Solar System Positioning System

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BIOGRAPHY

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ABSTRACT

Power-rich spacecraft envisioned in Prometheus initiative open up possibilities for long-range high-rate communication. A constellation of spacecraft on orbits several A.U. from the Sun, equipped with laser transponders and precise clocks can be configured to measure their mutual distances to within few cm. High on-board power can create substantial non-inertial contribution to the spacecraft trajectory. We propose to alleviate this contribution by employing secondary ranging to a passive daughter spacecraft.
Such constellation can form the basis of a navigation system capable of providing position information anywhere in the solar system with similar accuracy. Apart from obvious Solar System exploration implications, this system can provide robust reference for GPS and its successors.

INTRODUCTION

New developments in ion propulsion and fission power generation are likely to result in availability of gargantuan spacecraft with essentially unlimited power within a decade or two. Apart from obvious implications for Solar System exploration (despite recent programmatic setbacks for JIMO and Prometheus), these spacecraft and their successors will be available for suitable guest missions.

Available technology, such as Lunar Laser Ranging and Satellite Laser Ranging currently allow solving for the distance to a retro-reflector to within a few centimeters both for satellites in earth orbits and for the moon. In the case of lunar laser ranging, the reflected signal reaching earth is inversely proportional to the fourth power of distance. If the retro-reflector were to be replaced with an active transponder with a similar laser source, the signal intensity would fall off only as a square of the distance and the equivalent light intensities (1 photon per second) can be achieved for distances as large as 1000 A.U. Placing such transponders on a constellation of Sun-orbiting satellites effectively creates a solar-system-wide positioning system. Such system would be particularly attractive if the systematic errors associated with the ephemeris determination could be eliminated or substantially reduced. The impetus for the elimination of the systematics is clearly illustrated by the history of the Pioneer 10 and 11 spacecraft. Both of these spacecraft experienced an anomalous acceleration directed toward the Sun[1] of $\approx 9\times10^{-8} cm/s$. This acceleration could be expected from just ~60Watts of unisotropically emitted heat, out of budget of several thousands of Watts generated by two onboard Radioisotope Thermoelectric Generators (RTGs). This anomalous acceleration would be responsible for accumulated range error of ~3 meters in a day, if the ephemeris of the Pioneers was calculated using the JPL Solar System Ephemeris[2]. Other candidate contributions to the acceleration were the reaction form radio transmissions or leaking propellant. While unanticipated forces are still being favored for the observed acceleration, the possibility that the acceleration experienced by the Pioneer spacecraft are caused by unknown physics are substantially high to warrant follow-up tests. Design choices for the Pioneer 10 and 11 spacecraft are particularly fortuitous in that the acceleration was detected at all: later deep space missions either required more frequent orbit corrections or had highly asymmetric thermal emission or both.

RANGING SCHEME

As a possible mission to test for the anomalous acceleration the authors[3] proposed a two-step laser ranging scheme, where the position of a passive spacecraft is precisely tracked. The basic configuration of the proposed mission consists of a larger, powered spacecraft (mother spacecraft) and a smaller, passive proof mass equipped with retro-reflectors as shown in Figure 1. The ranging sequence is accomplished as follows. An earth-based or space-based interrogating station A transmits a ranging laser pulse. As the signal reaches the mother spacecraft B, two events are triggered. Spacecraft B transmits a ranging pulse back at A, and performs an independent ranging and attitude measurement to determine relative position of the proof mass. The time lapsed between the arrival of the pulse from A and transmission of the response pulse is

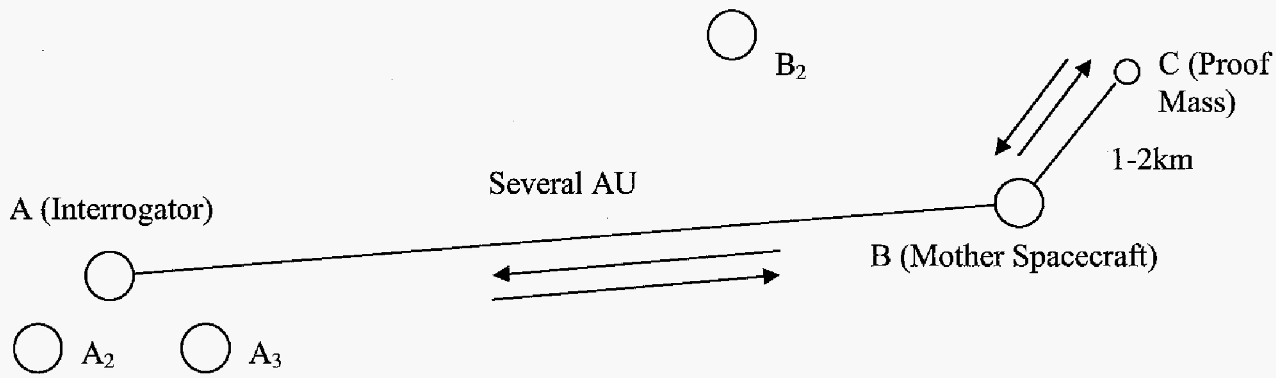

Figure 1. Basic ranging configuration. The interrogator A sends a laser light pulse toward the remote spacecraft B. B responds with a light pulse back and simultaneously measures the relative position of a passive, small proof mass C. The position information and the time lapsed between the arrival of the signal and re-transmission are reported back to A. Ranges for multiple interrogators to multiple spacecraft pairs can be determined.

measured with an onboard clock. That time and the position of the proof mass with respect to B are then transmitted to A. This sequence allows A to determine the range to the proof mass to within the resolution of the laser ranging scheme. The motion of the mother spacecraft B, while contributing to the distance between A and B, and to the relative position of proof mass C to B, is naturally subtracted in the calculation and therefore does not affect the measurement.

As part of preliminary analysis of the proposed scheme we calculated the magnitudes of several known contribution to the acceleration and ranging error. These include acceleration due to solar radiation, the uncertainty in our knowledge of the Earth's orbit, the clock error and the uncertainty in timing due to the width of the laser pulse. For simplicity, we assume that the trajectories of the spacecraft and that of the proof mass are in the orbital plane of the Earth, so that a two dimensional model is sufficient.

ERROR ESTIMATES

Error due to the determination of the angle to the proof mass

Determination of the range to the proof mass includes the projection of the distance between the mother spacecraft to the proof mass onto the line form the source (A) to the mother spacecraft. Therefore, the angle between the two needs to be known well enough to preserve the ranging accuracy. The lowest error would be achieved if the three objects lie on the same line. If the distance to the proof mass is less than a few kilometers, and the angle determination is achieved to better than $10^{-5} rad$, the ranging error attributable to the angle uncertainty is lower than a few centimeters.

Error in determining the distance from the proof mass to the center of the sun

From the distance between the proof mass and the Earth, it is possible to determine the distance from the proof mass to the sun using the JPL planetary ephemeris model. This process introduces additional errors due to the uncertainties in the model. How the error propagates within the model will require further studies. However, we expect the uncertainties in the model to continue to decrease with time, and also that the range data of the proposed constellation can be used to reduce this uncertainties even further. We do not expect this source of error to be significantly larger than a few centimeters.

Error due to Transmission Through Earth's Atmosphere

If the range is measured to an earth-based observatory, additional error can be expected from the variations in the atmospheric refraction. The lunar laser ranging experiments have already achieved centimeter-scale resolution with transmission through the atmosphere. Using the same error mitigation technique developed by lunar laser ranging experiments, we expect our range uncertainty to be limited by the width of the laser pulse, and not by disturbances in the atmosphere.

Error due to laser pulse width

YAG laser pulse width, 100*ps*, corresponds to a 3 cm pulse length. This error can be further mitigated by employing multiple detectors and analyzing the pulse shape.

Error due to clock

There is also an error associated with the clock at source A used to determine the round-trip time. With distances of a few AU, the round trip time can be as large as a few hours. At present, the best available clocks are the mercury-ion-trap clock and the cesium-fountain clock. The typical relative error is $(\delta t/t)_{clk} \approx 10^{-15}$ for time intervals exceeding a few hours. The uncertainty in the range due to clock error is $\delta d_{clk} = (c\tau/2)(\delta t/t)_{clk}$. The range error due to clock error in a single measurement with the round-trip time under 10 hours is of order 1cm. The clock error is determined entirely by the equipment at a single location (ground or interrogator satellite A).

Further significant improvements in clock technology and time transfer can be expected by the launch date, and by the time experiment is performed.

Error due to solar radiation pressure

Solar radiation pressure produces an average acceleration of $a = C_R \frac{A}{m}\frac{\Phi}{c}$, where A is the probe area, m is its mass, Φ is the solar flux ($\Phi = 1372.5$ *W/m²* at $1AU$), and C_R is a radiation pressure coefficient. The residual error will be due to variation of solar flux Φ and the radiation pressure coefficient C_R with time. The variation in Φ is about 0.2%, and it is in synchronization with the 11-year solar cycle[4]. The maximum occurs during periods of high solar activity when the number of sunspots changes at a relatively large rate. The solar flux is being measured continuously by several satellites. Alternatively, a radiometer can be installed on the mother spacecraft to measure the sun's radiation directly. Therefore the variations in Φ can reasonably be corrected to a level of 10 ppm.

The reflectivity of the retro-reflectors on the proof mass may change with time due to aging, resulting in a change in the value of C_R. Such possible changes must also be kept below a level of 10 ppm. Otherwise, the reflectivity must be measured periodically. The intensity of the reflected light measured during the laser ranging process contains information of the reflectivity. Obtaining the reflectivity data from laser ranging deserves further studies. Assuming that the variations in both Φ and C_R is 10^{-5} of their average values, and $C_R \approx 1.5$, the resultant error due to solar pressure is $\delta a = 7\times10^{-13}/r^2$ m/s², where r is in AU. While significant in long-term measurements such as check of the inverse-square law, this contribution can be corrected in real time for the purposes of ranging.

Error due to solar wind

Solar activity results in highly variable flux of particles (mostly hydrogen) away from the sun. Typical velocity of this Solar wind is 400km/s, and its typical density is 3 protons/cm² at 1AU. The contribution due to solar wind is $\delta a = C_W \frac{v^2 A\rho}{mr^2}$, where v is the particle velocity, A is the proof mass area and m is its mass, ρ is the mass density of the solar wind at Earth's orbit and r is the distance from the sun in AU. Solar wind coefficient C_W is of order 1. The calculated acceleration for the $10cm^2$, $1kg$ proof mass is $\delta a = 8\times10^{-12}/r^2$ *m/s²*, away from the sun. The variation of the particle flux and their energy distribution will affect the test. Other than during high solar activity period, this variation can be modeled to better than 1% of the average flux using measurements made by other satellites. Alternatively, an onboard particle detector can measure this flux to 1%. Therefore it is possible to correct the solar wind effect to below $10^{-14}\,m/s^2$ at ~ 3AU. This effect decreases with distance as $1/r^2$. Similarly to the radiation pressure discussion, slow variations in the acceleration due to solar wind can be taken into account by consistency checks.

Error due to charge buildup on the spacecraft and on the test mass

For a conducting sphere of radius R, the total charge q on it is related to its potential V by $V = q/(4\pi\varepsilon_o R)$, where $\varepsilon_o = 8.85\times10^{-12}\,C^2/Nm^2$. The electrostatic force will cause the proof mass to accelerate by an amount: $a = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1 q_2}{mr^2} = \frac{4\pi\varepsilon_o V_1 V_2 R_1 R_2}{mr^2}$. We model the spacecraft by a conducting sphere of 0.5 m radius, and the proof mass by a sphere of 0.05*m* radius, and both are charged to a typical value of 4kV. At 2*km* away, the acceleration on a 1*kg* proof mass is $1.1\times10^{-11}\,m/s^2$. This contribution to the acceleration can be measured by having the mother spacecraft orbit the proof mass. To avoid the necessity to measure or model the effects of charging while obtaining 10^{-14}*m/s²* of stray acceleration, the mother spacecraft must be discharged to less than 4*V*. The way to achieve this is the subject of further studies.

Error due to the charged proof mass moving in the magnetic field of the sun

The acceleration exerted by a magnetic field $\vec{B}$ on a charged body moving with a velocity $\vec{u}$ is $\vec{a} = q\vec{u}\times\vec{B}/m$. If the spacecraft is moving in the plane of the ecliptic, the tangential velocity of the proof mass will interact with the magnetic field of the sun, which is perpendicular to this plane, resulting in a radial acceleration. As the spacecraft leaves the Earth, it carries the tangential velocity of the orbit of the Earth with it. This tangential velocity has a magnitude of 30*km/s*. Therefore the largest acceleration on the proof mass is when it is at the vicinity of the Earth's orbit (~1*AU*), where the sun's magnetic field is also strongest. The magnitude of the sun's magnetic field changes from 1 to 37nT. It direction flips every 11 years, in synchronization with the solar cycle. When the proof mass is charged to 4*kV*, the charge on it is estimated to be $2.2\times10^{-8}\,C$. Assuming 37*nT* of magnetic field, the radial force on the proof mass is $2.5\times10^{-11}\,m/s^2$. If the voltage is reduced to 40V, then this acceleration can be reduced to $2.5\times10^{-13}\,m/s^2$. Since the $|\vec{B}| \propto 1/r^3$ and the tangential velocity is proportional to $1/r$ in order to conserve the angular momentum, we expect that acceleration falls off as $1/r^4$, and then beginning at $2.2AU$, the acceleration due to magnetic field is below $10^{-14} m/s^2$. Similar to the solar radiation and solar wind, this contribution to the acceleration can be modeled and subtracted.

Error due to gravitation and radiation from the mother spacecraft

The gravitational attraction from a $18000kg$ spacecraft produces acceleration of 3×10^{-13} m/s^2 at $2km$ distance, towards the spacecraft. On-board power source of $1MW$, isotropically radiated, produces acceleration of 1×10^{-12} m/s^2 away from the spacecraft at 2 km distance. Both sources can be modeled to better than 1%. Alternatively, thermal radiation can be directed away from the direction of the proof masses. The mass and power used in this calculation represent a worst-case projection for a thermal nuclear powered spacecraft. The actual distance to place the test masses will be determined once the mass and power of the mother spacecraft is known. Conventional spacecraft has 10 times smaller mass and 100 times smaller thermal radiation, making this effect negligible.

CONCLUSION

The discussion of optimal number and placement of satellites in the constellation used for Solar System Positioning System is beyond the scope of this paper. A validation hitchhiker mission on one of the future Prometheus spacecraft will be helpful not only in resolving the Pioneer anomaly and related fundamental science questions, but in confirming the possible utility of the laser-ranging constellation with passive proof masses for precise solar system navigation.

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